The Continuing Miracle of Information Storage Technology

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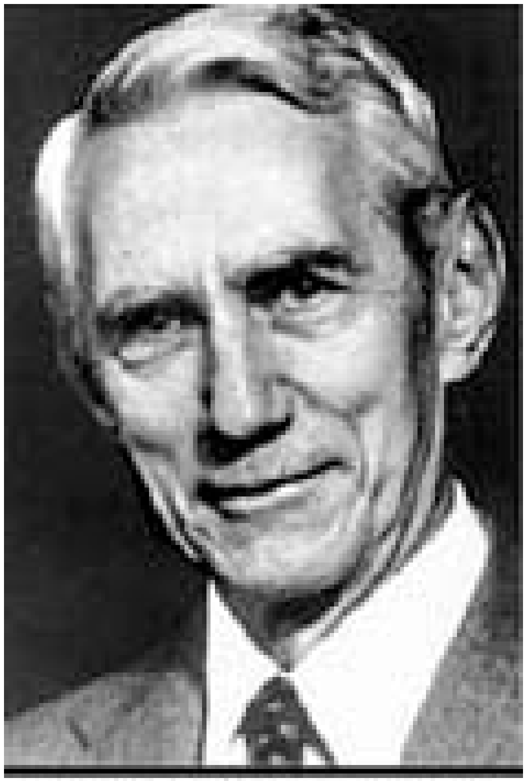
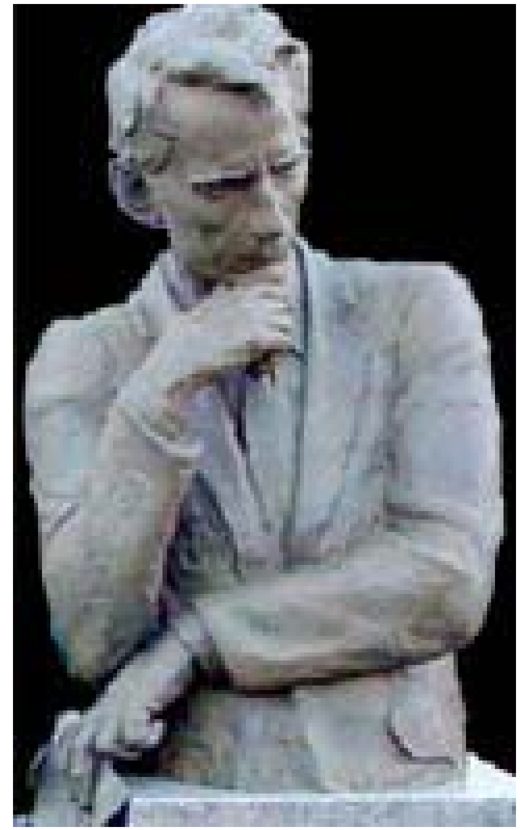
Outline

- **The Shannon Statue**
- **A Miraculous Technology**
- **Information Theory and Information Storage**
  - **A Tale of Two Capacities**
- **Conclusion**

Claude E. Shannon



Claude Elwood Shannon
1916 - 2001



Acknowledgments

- **For the statue - from conception to realization:**
  - **IEEE Information Theory Society**
  - **Prof. Dave Neuhoff (University of Michigan)**
  - **Eugene Daub, Sculptor**

- **For bringing it to CMRR:**
  - **Prof. Jack K. Wolf**

How Jack Did It

- **6 casts of the statue**
- **Spoken for:**
  1. **Shannon Park, Gaylord, Michigan**
  2. **The University of Michigan**
  3. **Lucent Technologies – Bell Labs**
  4. **AT&T Research Labs**
  5. **MIT**
- **Jack's idea: "6. CMRR"**

The Inscription

CLAUDE ELWOOD SHANNON

1916 – 2001

FATHER OF INFORMATION THEORY

HIS FORMULATION OF THE MATHEMATICAL THEORY OF COMMUNICATION PROVIDED THE FOUNDATION FOR THE DEVELOPMENT OF DATA STORAGE AND TRANSMISSION SYSTEMS THAT LAUNCHED THE INFORMATION AGE.

DEDICATED OCTOBER 16, 2001

EUGENE DAUB, SCULPTOR

Data Storage and Transmission

- **A data transmission system communicates information through space, i.e.,**

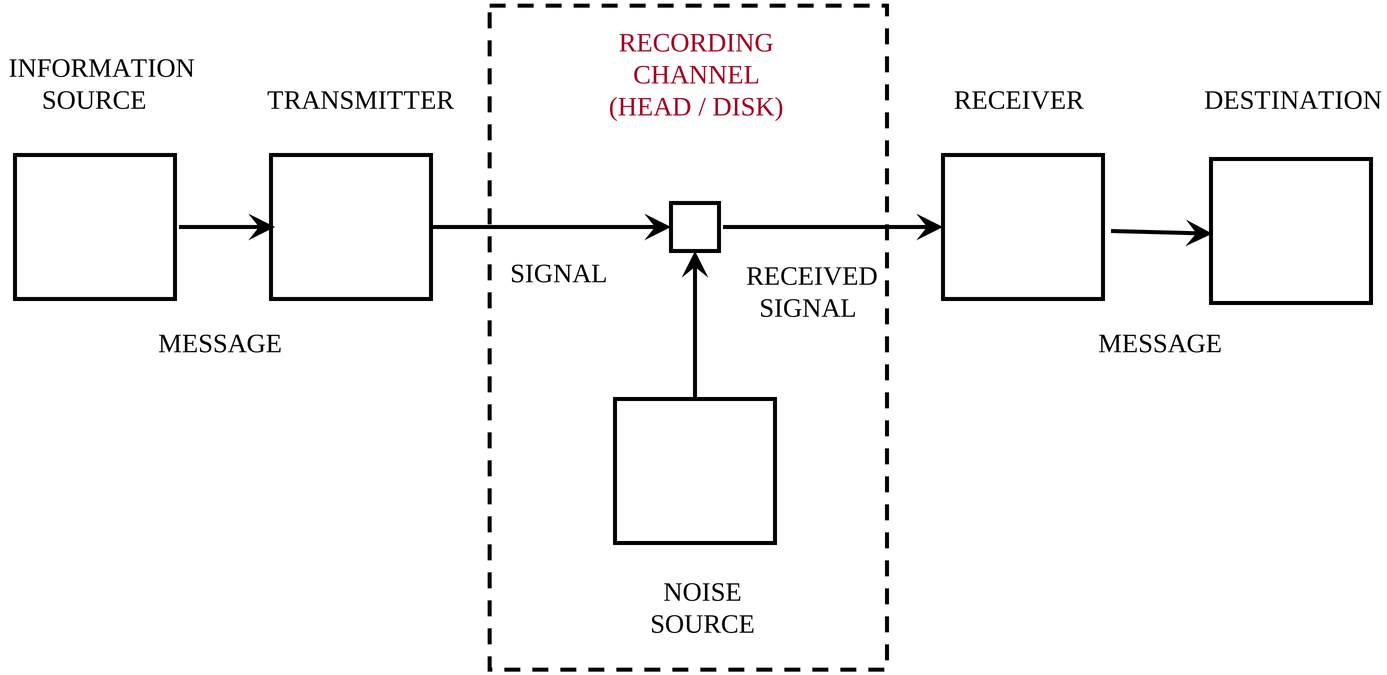
  “from here to there.”

- **A data storage system communicates information through time, i.e.,**

  “from now to then.”

[Berlekamp, 1980]

Figure 1
(for Magnetic Recording)

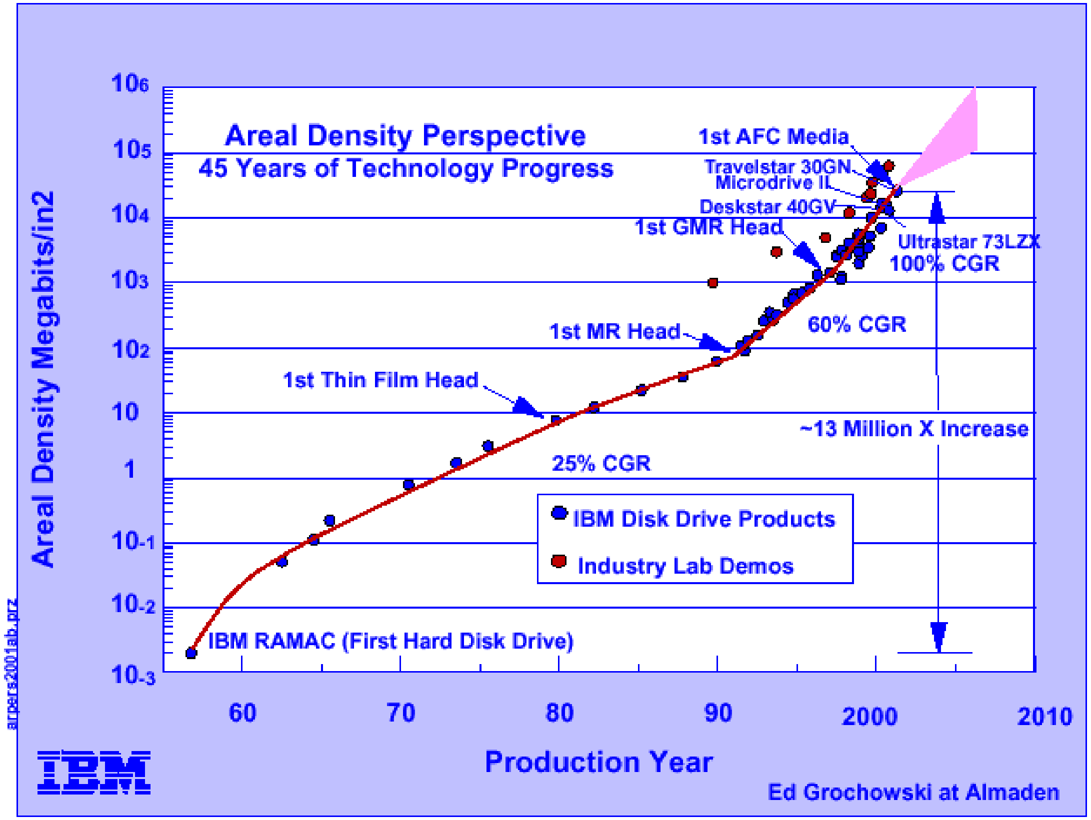


- **Binary-input**
- **Inter-Symbol Interference (ISI)**
- **Additive Gaussian Noise**

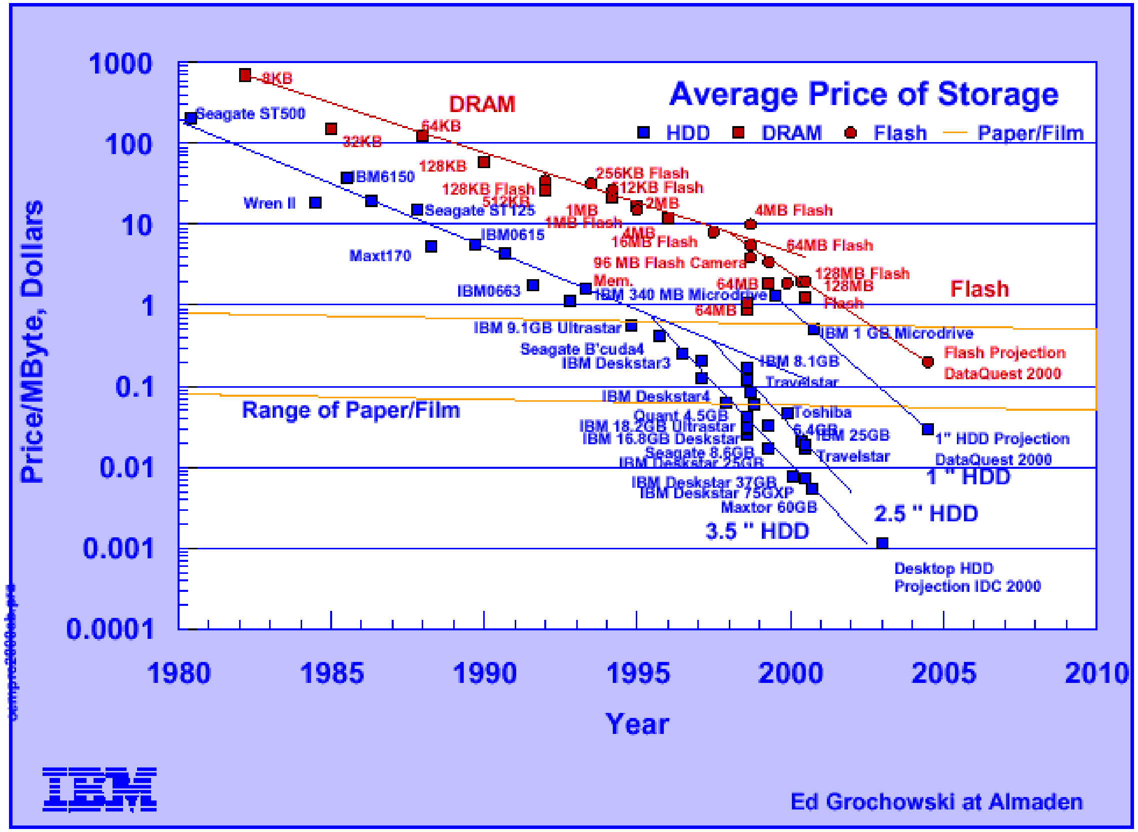
A Miraculous Technology

- **Areal Density Perspective – 45 Years of Progress**
- **Average Price of Storage**

Areal Density Perspective



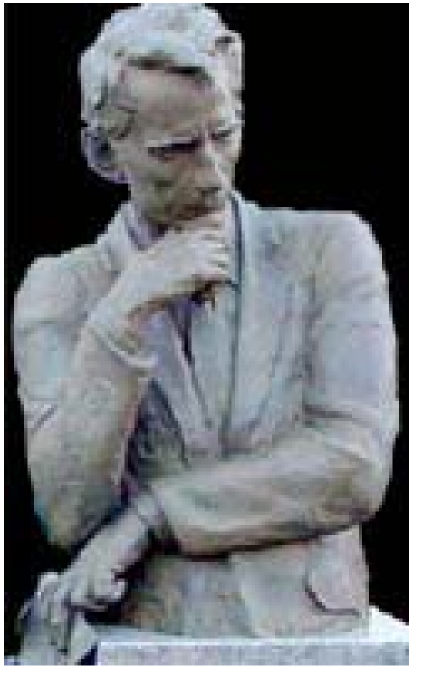
Average Price of Storage



The Formula on the "Paper"

Capacity of a discrete channel with noise [Shannon, 1948]

$$C = \mathrm{Max}\,(H(x) - H_y(x))$$



For noiseless channel, $H_y(x)=0$, so:

$$C = \mathrm{Max}\,H(x)$$

Gaylord, MI: C = W log (P+N)/N

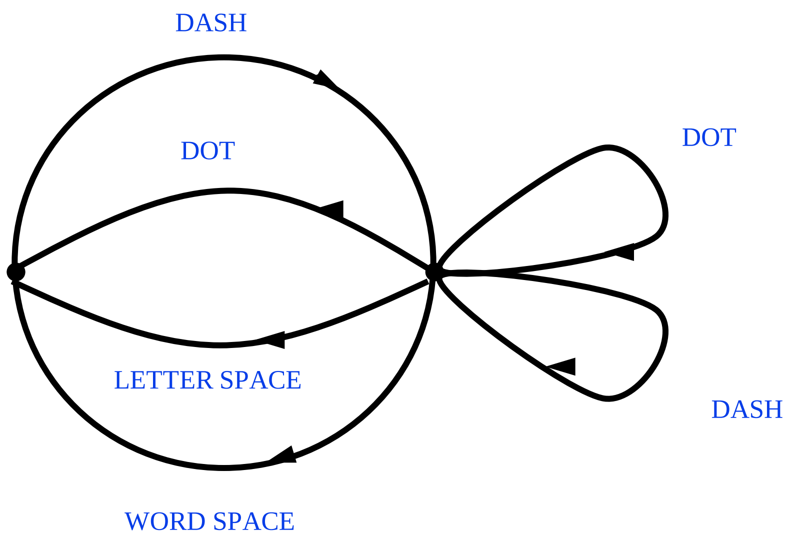
Bell Labs: no formula on paper

("H = – p log p – q log q" on plaque)

Discrete Noiseless Channels (Constrained Systems)

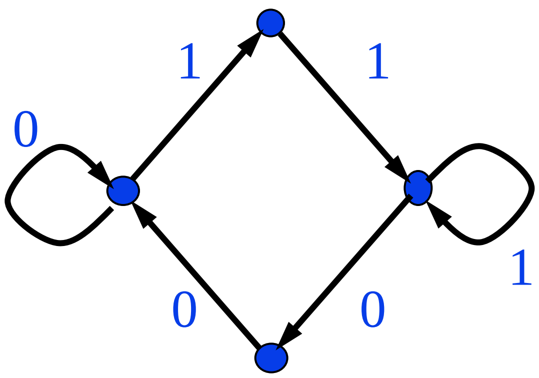
- **A constrained system S is the set of sequences generated by walks on a labeled, directed graph G.**

Telegraph channel constraints [Shannon, 1948]



Magnetic Recording Constraints

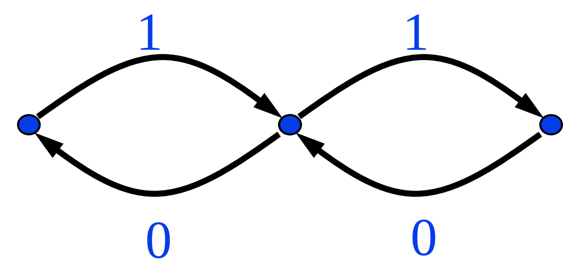
Runlength constraints
("finite-type": determined by finite list F of forbidden words)



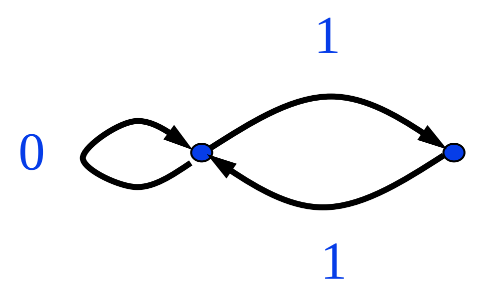
Forbidden words F={101, 010}

Spectral null constraints
("almost-finite-type")

Biphase



Even



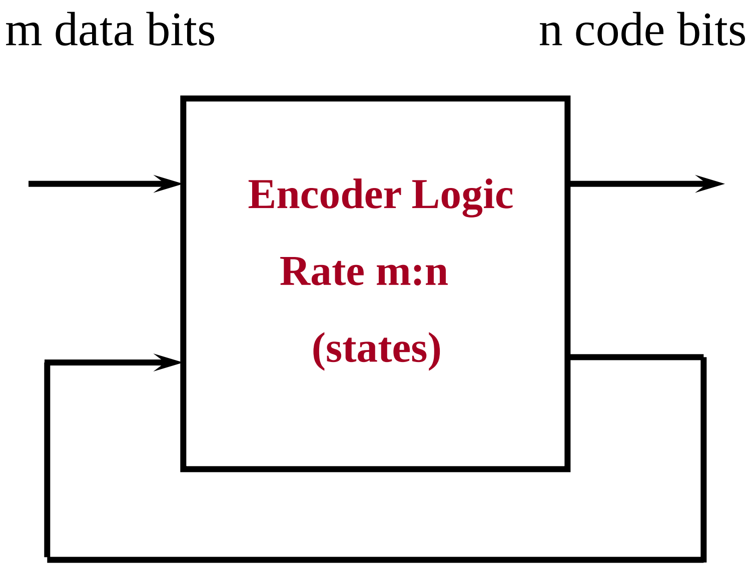
Practical Constrained Codes

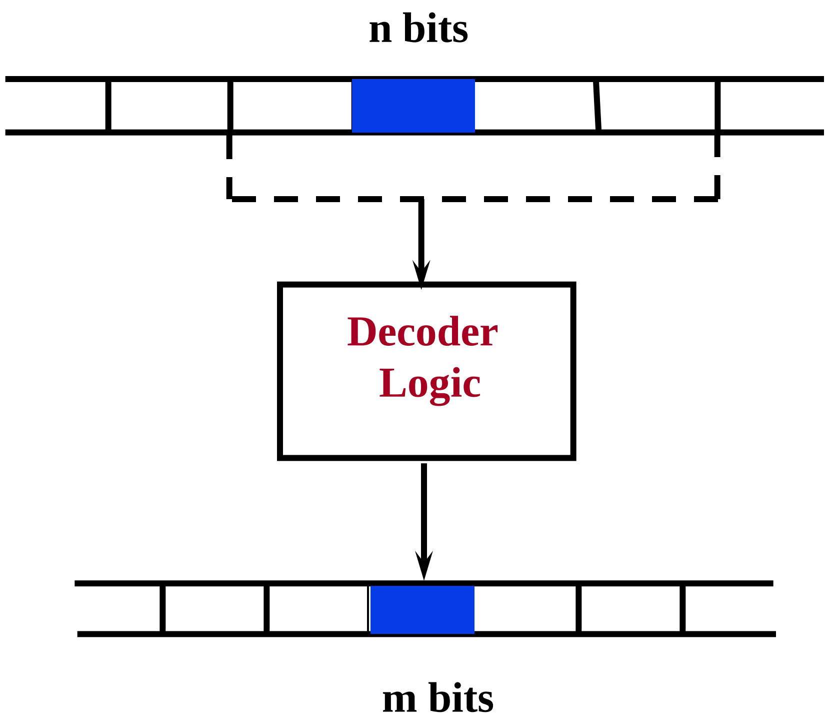
Finite-state encoder

(from binary data into S)

Sliding-block decoder

(inverse mapping from S to data)





We want: high rate R=m/n
low complexity

Codes and Capacity

- **How high can the code rate be?**
- **Shannon defined the capacity of the constrained system** S:

$$C = \lim_{n \to \infty} \frac{1}{n} \log N(S,n)$$

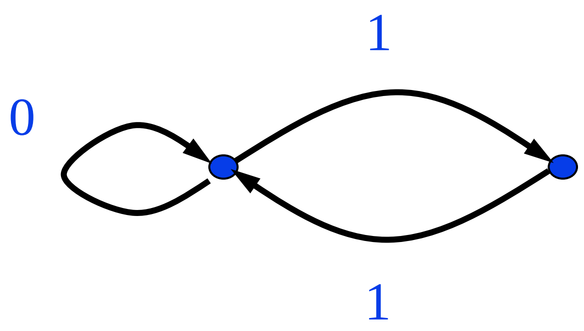
where $N(S,n)$ **is the number of sequences in** S **of length** n**.**

Theorem [Shannon,1948] : If there exists a decodable code at rate $R = m/n$ **from binary data to** S**, then** $R \le C$**.**

Theorem [Shannon,1948] : For any rate $R = m/n < C$ **there exists a block code from binary data to** S **with rate** $km{:}kn$**, for some integer** $k \ge 1$**.**

Computing Capacity: Adjacency Matrices

- **Let A_G be the adjacency matrix of the graph G representing S.**



$$A_G = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

- **The entries in $A_G{}^n$ correspond to paths in *G* of length n.**

Computing Capacity (cont.)

- Shannon showed that, for suitable representing graphs G ,

$$C = \log\ \rho(A_G)$$

where $\rho(A_G) = \max\{\, |\lambda| : \lambda \text{ is an eigenvalue of } A_G \}$, i.e., the spectral radius of the matrix A_G .

- Assigning "transition probabilities" to the edges of G, the constrained system S becomes a Markov source x, with entropy $H(x)$. Shannon proved that

$$C = \max\ H(x)$$

and expressed the maximizing probabilities in terms of the spectral radius and corresponding eigenvector of A_G .

Constrained Coding Theorems

- **Stronger coding theorems were motivated by the problem of constrained code design for magnetic recording.**

  Theorem[Adler-Coppersmith-Hassner, 1983]

  Let S **be a finite-type constrained system. If** $m/n \leq C$, **then there exists a rate** $m{:}n$ **sliding-block decodable, finite-state encoder.**

  (Proof is constructive: state-splitting algorithm.)

  Theorem[Karabed-Marcus, 1988]

  Ditto if S **is almost-finite-type.**

  (Proof not so constructive…)

Distance-Enhancing Codes for Partial Response Channels

- **Beginning in 1990, disk drives have used a technique called partial-response equalization with maximum-likelihood detection, or PRML. In the late 1990's, extensions of PRML, denoted EPRML and EEPRML were introduced.**
- **The performance of such PRML systems can be improved by using codes with "distance-enhancing" constraints.**
- **These constraints are described by a finite set D of "forbidden differences," corresponding to differences of channel input sequences whose corresponding outputs are most likely to produce detection errors.**

Codes that Avoid Specified Differences

- **The difference between length-*n* binary words *u* and *v* is**

$$u - v = (u_1 - v_1, \ldots, u_n - v_n) \in \{-1, 0, 1\}^n$$

- **A length-*n* code avoids *D* if no difference of codewords contains any string in *D*.**

- **Example:** $D = \{++, +-\}$

Length-2 code: $C_2 = \{u, v\} = \{00, 10\}$

$$u - v = (-0)$$

Capacity of Difference Set D
[Moision-Orlitsky-Siegel]

- **How high can the rate be for a code avoiding** *D***?**
- **Define the capacity of the difference set** *D***:**

$$cap(D) = log\left[\lim_{n\to\infty} \left(\delta_n(D)\right)^{1/n}\right],$$

where $\delta_n(D)$ is the maximum number of codewords in a (block) code of length *n* **that avoids** *D***.**

- **Problem: Determine** $cap(D)$ **and find codes that achieve it.**

Computing cap(D): Adjacency Matrices

- **Associate to** D **a set of graphs and corresponding set** $\Sigma(D)$ **of adjacency matrices reflecting disallowed pairs of code patterns:**

$$\Sigma = \Sigma(D) = \{A_i : i = 1,\ldots,k\}$$

- **Consider the set of** n**-fold products of matrices in** Σ **:**

$$\Sigma^n = \left\{ \prod_{j=1}^{n} B_j : B_j \in \Sigma \right\}$$

- **Each product corresponds (roughly) to a code avoiding** D**.**

Generalized Spectral Radius $\rho(\Sigma)$

- **Define**

$$\rho_n(\Sigma) = sup\left\{ \rho(A) : A \in \Sigma^n \right\},$$

the largest spectral radius of a matrix in Σ^n.

- **The generalized spectral radius of Σ is defined as:**

$$\rho(\Sigma) = \lim_{n\to\infty} sup\left[\rho_n(\Sigma)\right]^{1/n}$$

[Daubechies-Lagarias,1992], cf. [Rota-Strang, 1960]

Computing cap(D) (cont.)

Theorem[Moision-Orlitsky-Siegel, 2001]

For any finite difference set D**,**

$$cap(D) = log\ \rho(\Sigma(D))\ .$$

Recall formula for the capacity of a constrained system S

$$C = log\ \rho(A_G)\ .$$

- **Computing** $cap(D)$ **can be difficult, but a constructive bounding algorithm has yielded good results.**

A Real Example: EEPRML

- **Codes can improve EEPRML performance by avoiding**

$$D = \{0 + - + 0\}$$

- **Codes satisfying the following constraints avoid** D**:**

  - **F={101,010}** **C=0.6942…**
  - **F={101}** **C=0.8113…**
  - **F={0101,1010}** **C=0.8791…** **MTR [Moon,1996]**

- **What is** $cap(D)$**, and are there other simple constraints with higher capacity that avoid** D **?**

EEPRML Example (cont.)

- **For** $D = \{0 + - + 0\}$ **,**

$$0.9162 \leq cap(D) < 0.9164.$$

- **The lower bound, conjectured to be exactly** $cap(D)$**, is achieved by the "time-varying MTR (TMTR)" constraint, with finite periodic forbidden list:**

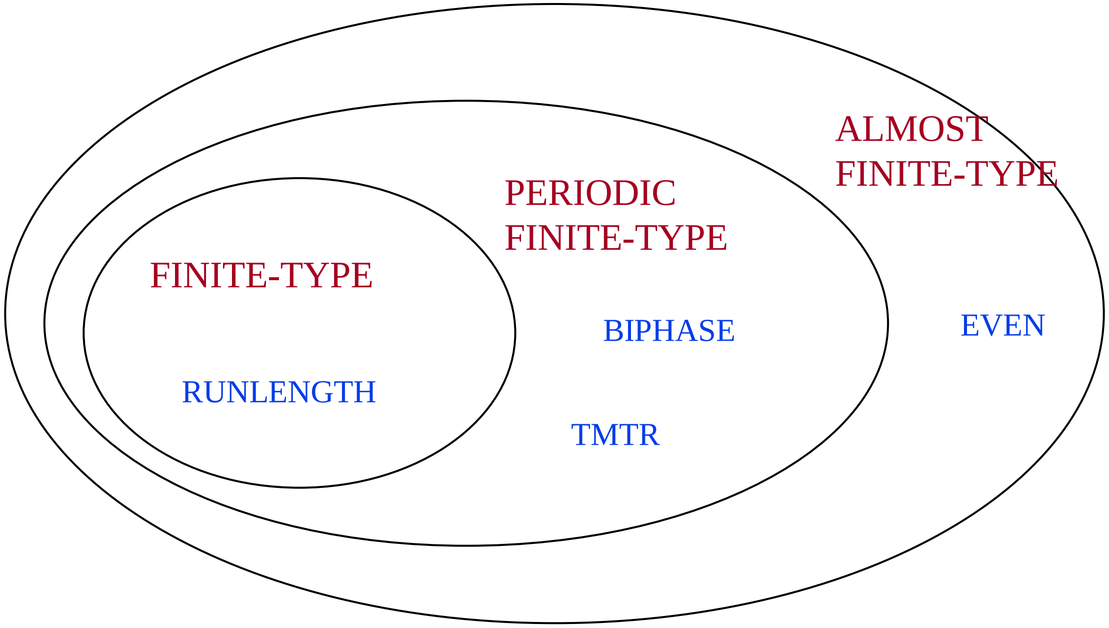
$$F = \{1010^{odd}, 0101^{odd}\}.$$

- **Rate 8/9, TMTR code has been used in commercial disk drives [Bliss, Wood, Karabed-Siegel-Soljanin].**

Periodic Finite-Type Constraints

- **The TMTR (and biphase) constraint represent a new class of constraints, called "periodic finite-type," characterized by a finite set of periodically forbidden words.**

  [Moision-Siegel, ISIT 2001]



Other Storage-Related Research

- **Page-oriented storage technologies, such as holographic memories, require codes generating arrays of bits with 2-dimensional constraints. This is a very active area of research.**

  [Wolf, Shannon Lecture, ISIT 2001]

- **There has been recent progress related to computing the capacity of noisy magnetic recording (ISI) channels,**

$$C = \mathrm{Max}\,(H(x) - H_y(x)) .$$

[Arnold-Loeliger, Arnold-Vontobol, Pfister-Soriaga-Siegel, Kavcic, 2001]

Conclusion

- **The work of Claude Shannon has been a key element in the "miraculous" progress of modern information storage technologies.**

- **In return, the ongoing demand for data storage devices with larger density and higher data transfer rates has "miraculously" continued to inspire new concepts and results in information theory.**